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Real-Time Optimal State Feedback Control for Tethered Subsatellite System

Kentaroh Kokubun,* Seiichi Anazawa,† and Hironori A. Fujii‡
Tokyo Metropolitan Institute of Technology, Tokyo 191, Japan

Introduction

ALTHOUGH many papers are devoted to the study of control problems of the tethered subsatellite systems, only a few papers[1–4] treat the control problems as optimization problems. Bainum and Kumar[1] introduced an optimal-control law based on an application of the linear quadratic regulator (LQR) to the tethered satellite system. They apply the tension control law, employing feedback of tether length, in-plane angle, their rates, and the commanded length for the deployment and retrieval and stationkeeping of the subsatellite; the performance index for the optimization problems is set to correct deviations from the instantaneous equilibrium state at each time. No and Cochran[2] show that the LQR can be applied to control stationkeeping and maneuvering of the tethered subsatellite by utilizing the atmospheric aerodynamics, and the performance index for the optimization problem is set to correct deviations from the reference trajectory and to converge into the desired state at the final time. Netzer and Kane[3] obtain an optimal-length law of the tethered subsatellite system with the performance index involving penalties on the terminal values of the states as well as on the states and controllers throughout the maneuver, and the optimal solution is used as a nominal path to follow. They show that the LQR can be applied to derive tracking-type feedback control law to decrease the deviation of the subsatellite from the nominal path by using the thrusters. On the assumption that the control force is only the tether tension and that no control force or energy dissipation exists for motion perpendicular to the tether line, Fujii and Anazawa[4] obtain an optimal path in the sense that the time integral of squared tension plus squared in-plane angle is the performance index with inequality constraints on the control force. The Lyapunov approach is applied to derive the tracking-type feedback control law to follow the optimal path in Ref. 4. It is generally difficult to obtain the optimal path by solving the two-point boundary-value problem. To overcome this difficulty, Ohtsuka and Fujii[5] adopted the stabilized continuation method,[6] which converts the two-point boundary-value problem into the initial-value problem. Combining this method with the receding horizon control method,[7] they have succeeded in developing the real-time optimal state feedback controller.[8] This Note applies this real-time optimal state feedback control to the deployment and retrieval control problem of the tethered subsatellite and to determine the effective application of the control.

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*Graduate Student, Department of Aerospace Engineering; currently Master of Engineering, Ship Research Laboratory, Mitaka, Tokyo 181, Japan.

†Graduate Student, Department of Aerospace Engineering; currently Master of Engineering, Mitsubishi Heavy Industries, Ltd., Minato-ku, Nagoya 455, Japan.

‡Professor, Department of Aerospace Engineering. Associate Fellow AIAA.

Tethered Subsatellite System

A tethered subsatellite connected to the Shuttle is illustrated in Fig. 1. The center of attraction is denoted by O and the center of mass of the Shuttle by C. The orthogonal axes X and Y are defined along OC and along the orbital velocity vector, respectively, both originating at C. The parameters m, l, and Θ denote mass of the subsatellite, length of the tether, and rotational angle of the subsatellite in the orbital plane, respectively.

In this study, the following assumptions are made:

1) The tether has no mass, and thus its flexibility is ignored.

2) The mass of the subsatellite is sufficiently small with respect to that of the Shuttle that C always remains in its nominal orbit.

3) The external force affecting the motion is only the gravitational force caused by O. The orbit is circular, and only motion in the orbital plane is considered.

4) The control force acts only along the tether through tension $\tilde{T}$, and no control force or energy dissipation exists for motion perpendicular to the tether line.

On the above assumptions, the dimensionless equations of motion are obtained as follows:

$$\Lambda'' - \Lambda\Theta'^2 - 2\Lambda\Theta' - 3\Lambda\cos^2\Theta = -\hat{T} \tag{1}$$

$$\Theta'' + 2(\Lambda'/\Lambda)\Theta' + 3\sin\Theta\cos\Theta + 2(\Lambda'/\Lambda) = 0 \tag{2}$$

where $()' = \mathrm{d}()/\mathrm{d}\tau$, where $\tau = \Omega t$, t is time, and Ω is the angular velocity of the Shuttle in its orbit; $\Lambda = l/L$, where L is the desired length for the deployment; and $\hat{T} = \tilde{T}/(m\Omega^2 L)$.

Real-Time Optimal State Feedback Control for the Tethered Subsatellite System

Solution of the optimal control problem, in general, is obtained by solving the two-point boundary-value problem, which consists of Euler–Lagrange equations derived from the stationary condition of augmented performance index with multiplier functions.[9] Numerical calculation is necessary to solve the two-point boundary-value problem with iterative process because it is difficult to obtain an optimal solution analytically. Several methods have been proposed to solve the optimal control problem.[10–12] These methods require a suitable candidate of the initial guess because the accuracy of convergence into the optimal solution depends on the initial guess. Such a solution is, however, difficult to find, and an unsuitable solution makes it difficult to satisfy the boundary conditions. To overcome this difficulty, the two-point boundary-value problem is converted into the initial-value problem by the continuation method[5,6] in this Note. The following performance index is considered for the tethered subsatellite system:

$$J = \frac{1}{2}c_x(x - x_f)\bigg|_{\tau = T_f}^{2} + \frac{1}{2}R\int_0^{T_f}\hat{T}^2\,\mathrm{d}\tau \tag{3}$$

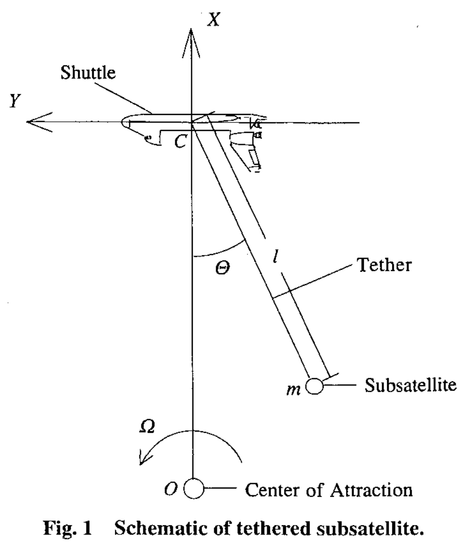

Fig. 1 Schematic of tethered subsatellite.

where $x^T = (\Lambda \;\; \Lambda' \;\; \Theta \;\; \Theta')$ is the state variable, T_f is the dimensionless desirable terminal time, x_f is the desirable terminal state variable, and c_x and R are positive weighting coefficients on the terminal state variable and control input, respectively.

Because the optimal solution obtained under the condition of free or fixed terminal time does not guarantee robustness, the receding horizon control[7,8] is required to let the actual solution converge into the optimal solution and to reduce deviation from the optimal solution. The optimality condition of the receding horizon control is obtained as a function of λ^*, x^* and the variable terminal time T as

$$F[\lambda(0)^*, x(0)^*, T] = \lambda(T)^* - \varphi_x^T[x(T)^*] = 0 \tag{4}$$

where λ is the Lagrange multipliers, φ is the equality constraint, and $()^*$ denotes a variable in the optimal control problem that distinguishes it from its correspondence in the actual system and $x^*(\tau) = x(t + \tau)$. In this study, time t is chosen as the continuation parameter, and the variable terminal time T is defined as a function of time t:

$$T(t) = T_f[1 - \exp(-\alpha \times t)] \tag{5}$$

where α is a positive constant. It is clear that $T(t)$ is equal to zero at $t = 0$ and converges to T_f as time t increases. The optimal problem is solved with the initial values of multiplier functions:

$$\lambda^*(0) = \lambda^*(T) = \varphi_x^T[x^*(T)] = \varphi_x^T[x^*(0)] \qquad (\text{at } t = 0) \tag{6}$$

The inequality constraints of the dimensionless tension are introduced as follows:

$$\hat{T}_{\min} \leq \hat{T} \leq \hat{T}_{\max} \tag{7}$$

The lower bound of the constraints is set, because the tether becomes slack if the tension is less than zero. The upper bound is also set, because the tolerable tension is limited for the existing material. The optimal condition for the control force is modified under the inequality constraint as follows:

$$\hat{T} = \arg[\min H(x, \lambda, \hat{T})] \tag{8}$$

where the Hamiltonian H is minimized by the optimal control force.

Results of Numerical Simulation

Numerical results are presented for the case in which the Shuttle is assumed to follow a circular orbit with a radius of 6600 km and an orbital velocity of 1.178×10^{-3} rad/s. The desired length for the deployment L is assumed to be 100 km. Mass of the subsatellite m is, however, not specified in the numerical simulation because the same control response of the system is obtained by modulating the weighting coefficients, which correspond to the prescribed value of the mass of the subsatellite.

The initial and terminal states of the deployment phase are given as follows:

$$\Lambda = 0.1, \quad \Lambda' = 0.085, \quad \Theta = 0, \quad \Theta' = 0 \qquad \text{(initial state)} \tag{9}$$

$$\Lambda = 1, \quad \Lambda' = 0, \quad \Theta = 0, \quad \Theta' = 0 \qquad \text{(terminal state)} \tag{10}$$

The deployment phase is set as $c_\Lambda = 800$, $c_{\Lambda'} = 450$, $c_\Theta = 1500$, $c_{\Theta'} = 100$, $R = 1$, $\alpha = 0.5$, $\zeta = 500$, $T_f = 0.02$, $\hat{T}_{\max} = 4$, and $\hat{T}_{\min} = 0$, where ζ is a positive constant to attenuate error.[8] Dimensionless equation $\hat{T}_{\max} = 4$ corresponds to dimensional equation $\tilde{T}_{\max} = 0.5551m$ [N] from $\hat{T} = \tilde{T}/(m\Omega^2 L)$. Therefore, it is obvious that $\tilde{T}_{\max}$ is identified by specifying the mass of the subsatellite.

Initial and terminal states for the retrieval phase are given as follows:

$$\Lambda = 1, \quad \Lambda' = 0, \quad \Theta = 0, \quad \Theta' = 0 \qquad \text{(initial state)} \tag{11}$$

$$\Lambda = 0.1, \quad \Lambda' = 0, \quad \Theta = 0, \quad \Theta' = 0 \qquad \text{(terminal state)} \tag{12}$$

The retrieval phase is set as $c_\Lambda = 2500$, $c_\Lambda = 450$, $c_\Theta = 1200$, $c_{\Theta'} = 40$, $R = 1$, $\alpha = 0.5$, $\zeta = 500$, $T_f = 0.02$, $\hat{T}_{\max} = 4$, $\hat{T}_{\min} = 0$. Figure 2 shows the time histories of the dimensionless length of the tether, rotational angle of the subsatellite in the orbital plane, and dimensionless tension during deployment and retrieval phases. It is seen that it takes about 400 min for the subsatellite to converge into the terminal state in both phases. The real-time optimal state feedback control is implemented successfully for control of both deployment and retrieval phases and has excellent features such as real-time feedback and nonrecursive process in the numerical calculation, compared with existing control methods.[3,4]

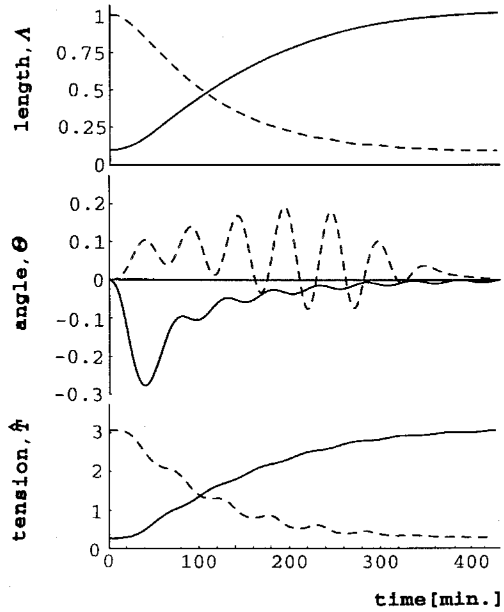

Fig. 2 Time histories of the dimensionless length of the tether, rotational angle of the subsatellite, and tension for the deployment and retrieval phases: ——, deployment and - - -, retrieval.

The real-time optimal state feedback control can be applied to the control problem of the dynamic system described by the ordinary differential equations. It is anticipated that the present control scheme can be applied to more realistic dynamics, such as tether flexibility, after discretizing the tether by a lumped-mass model. This is an area deserving additional work.

Conclusions

Real-time optimal state feedback control is applied successfully to a control problem of the tethered subsatellite system. The stabilized continuation method converts the two-point boundary-value problem into the initial-value problem, and the optimal state feedback controller is derived from the receding horizon control method. Results of numerical simulation show that the subsatellite is well controlled for the terminal state during deployment and retrieval phases.

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Regional Pole Placement Method for Discrete-Time Systems

Hardev Singh,* D. Subbaram Naidu,† and Kevin L. Moore‡

Idaho State University, Pocatello, Idaho 83209-8060

I. Introduction

THE dynamic characteristics of a linear system are influenced by the location of its poles. In general, it is sufficient that poles be placed in a suitable region of the complex plane, instead of placing them at their respective exact positions. The regional pole placement (RPP) technique was studied for continuous-time systems for placing the eigenvalues to the left of sector for a well-damped response.[1] The problem of designing feedback gains to optimally place the closed-loop poles of a discrete-time system has been investigated using a linear quadratic regulator (LQR) to obtain a linear state feedback law guaranteeing that the closed-loop poles lie inside a circle with given radius and centered at a distance on the real axis.[2,3] However, there appears to be no attempt made so far for the problem of placing the eigenvalues within a logarithmic spiral in the z plane for discrete-time systems.

In this Engineering Note, for discrete-time systems, a novel method of placing the eigenvalues corresponding to the region within a logarithmic spiral (which actually translates into a well-damped continuous-time system) is developed. Essentially, the method proposes a simple technique of fitting a circle within a logarithmic spiral.

II. LQR Theory for RPP in Discrete-Time Systems

Consider a linear, time-invariant, discrete-time controllable system

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

where x is an $n \times 1$ state vector, u is an $r \times 1$ control vector, and A and B are $n \times n$ and $n \times r$ constant matrices, respectively. We formulate an optimal control problem such that the optimal control minimizes a specified performance index while at the same time places the closed-loop poles inside a circular region. Given the plant dynamics of Eq. (1) and the performance index

$$J = \sum_{k=0}^{\infty} \left(\frac{1}{\alpha}\right)^{2k} [x^T(k)Qx(k) + u^T(k)Ru(k)] \tag{2}$$

where $Q = Q^T \geq 0$ and $R = R^T > 0$, it is well known[3] that the optimal control that minimizes Eq. (2) has closed-loop poles inside a circle centered at the origin with radius $r = \alpha$. Now, if

$$\bar{x}(k+1) = A_\beta \bar{x}(k) + B\bar{u}(k), \qquad A_\beta = A - \beta I \tag{3}$$

then the optimal control that minimizes

$$J = \sum_{k=0}^{\infty} \left(\frac{1}{\alpha}\right)^{2k} [\bar{x}^T(k)Q\bar{x}(k) + \bar{u}(k)^T R\bar{u}(k)] \tag{4}$$

has all of the closed-loop poles inside the region of the circle (of radius α and center β), as shown in Fig. 1. Thus, the problem reduces to finding a performance index so that the resultant optimal control subject to Eq. (1) is equivalent to the optimal control that minimizes Eq. (4) subject to Eq. (3). This problem can be further reduced to a standard LQR problem. Indeed, if we let

$$\hat{x}(k) = (1/\alpha)^k \bar{x}(k), \qquad \hat{u}(k) = (1/\alpha)^k \bar{u}(k) \tag{5}$$

$$\hat{A} = (1/\alpha)A_\beta, \qquad \hat{B} = (1/\alpha)B \tag{6}$$

then the dynamical equation (3) becomes

$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}\hat{u}(k) \tag{7}$$

and the performance index of Eq. (4) can be written as

$$J = \sum_{k=0}^{\infty} [\hat{x}(k)Q\hat{x}(k) + \hat{u}(k)R\hat{u}(k)] \tag{8}$$

The minimization problem with plant dynamics of Eq. (1) and the performance index of Eq. (2) is reduced to a standard LQR problem with plant dynamics of Eq. (7) and the performance index of Eq. (8). The optimal control law that minimizes Eq. (8) subject to the constraint of Eq. (11) is

$$\hat{u}(k) = -F\hat{x}(k) \tag{9}$$

where

$$F = [R + \hat{B}^T P \hat{B}]^{-1} \hat{B}^T P \hat{A} \tag{10}$$

and P is the symmetric, positive-definite solution of the algebraic Riccati equation

$$P = Q + \hat{A}^T P \hat{A} - \hat{A}^T P \hat{B} [R + \hat{B}^T P \hat{B}]^{-1} B^T P \hat{A} \tag{11}$$

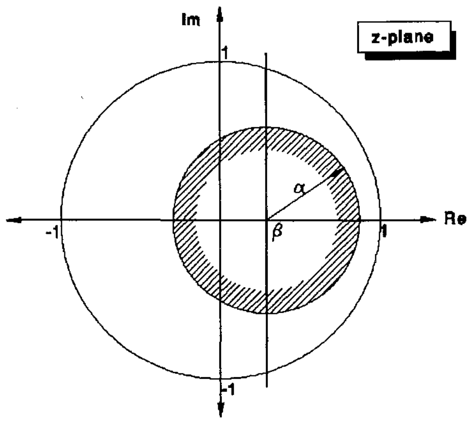

Fig. 1 Regional pole placement in discrete-time system.

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*Graduate Student, Measurement and Control Engineering Research Center, College of Engineering, P.O. Box 8060.

†Professor, Measurement and Control Engineering Research Center, College of Engineering, P.O. Box 8060. Associate Fellow AIAA.

‡Associate Professor, Measurement and Control Engineering Research Center, College of Engineering, P.O. Box 8060.